

Physics (Fields and Further Mechanics)Further Mechanics● **Momentum Concepts**

- Force is the rate of change of momentum,

$$F = \frac{\Delta(mv)}{\Delta t}$$

- The impulse of a force is:
 - The force multiplied by the time it acts for
 - The change in momentum it causes
- Impulse

$$F \Delta t = \Delta(mv)$$

- The area under a force-time graph is the change in momentum
- In a system of interacting particles, the total momentum remains constant, provided no external resultant force acts in the system
- An **elastic collision** is one where there is no loss of kinetic energy
- An **inelastic collision** occurs where the colliding objects have less kinetic energy after the collision than before the collision
 - Two cars colliding at low speeds is almost perfectly elastic if no damage is done. If damage is done then the collision is not elastic
- In an explosion the total momentum remains at zero
 - The two objects move away from one another with equal momentums in opposite directions

$$m_a v_a = - m_b v_b$$

● **Circular Motion**

- Motion at a constant velocity in a circular path implies there is an acceleration and therefore a centripetal force
- Angular speed

$$\omega = \frac{v}{r} = 2\pi f$$

- Centripetal acceleration

$$a = \frac{v^2}{r} = \omega^2 r$$

- Centripetal force

$$F = \frac{mv^2}{r} = m\omega^2 r$$

● **Simple Harmonic Motion**

- Simple harmonic motion is oscillating motion in which acceleration is
 - Proportional to the displacement
 - Always acts in the opposite direction to the displacement

- The simple harmonic motion equation

$$a = -\omega^2 x$$

- The displacement at time t is

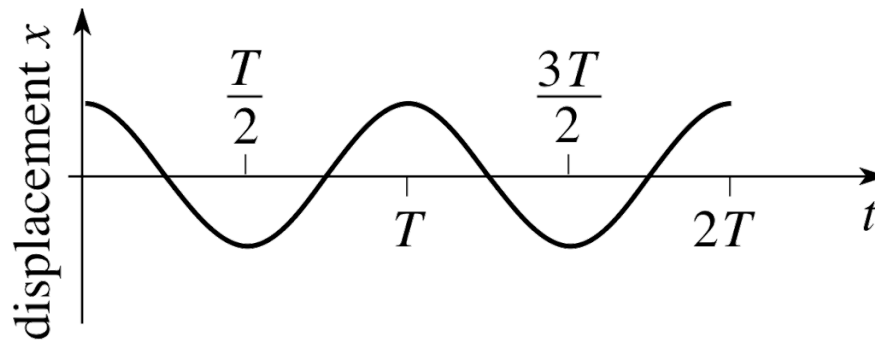
$$x = A \cos(\omega t)$$

where A is the amplitude

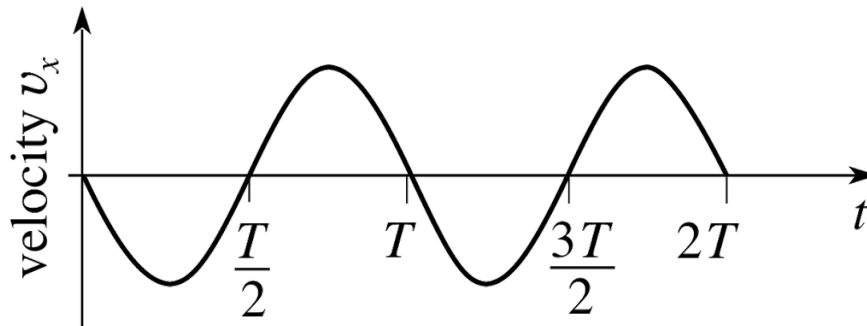
- The velocity of the motion at displacement x is

$$v = \pm 2\pi f \sqrt{A^2 - x^2}$$

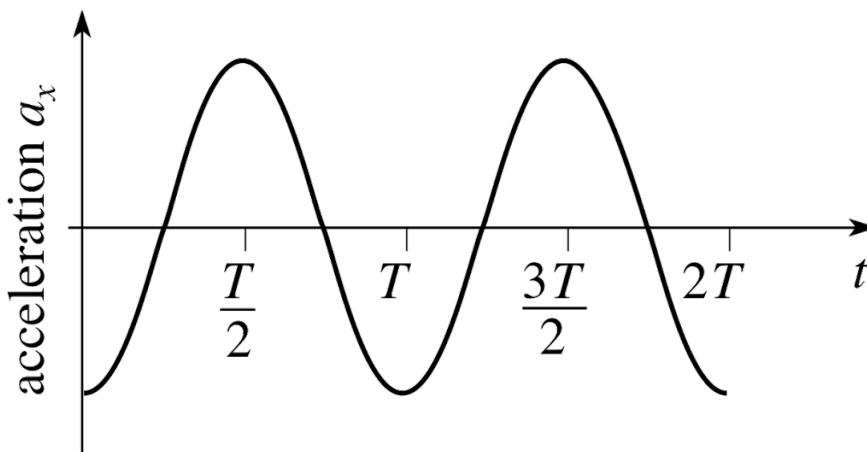
- Graph of displacement, velocity and acceleration against time



(a)



(b)



(c)

- Velocity is the gradient of the displacement graph
- Acceleration is the gradient of the velocity graph
- The maximum velocity is given by

$$v_{max} = 2\pi f A$$

- The maximum acceleration is given by

$$a_{max} = (2\pi f)^2 A$$

- **Simple Harmonic Systems**

- Mass-Spring system

- Factors that affect the frequency of oscillations of a loaded spring
 - Adding extra mass - Increases the inertia of the system, results each cycles being slower
 - Using weaker springs - The restoring force will be lower, resulting in each cycle being slower
- The period of the cycle will take

$$T = 2\pi\sqrt{\frac{m}{k}}$$

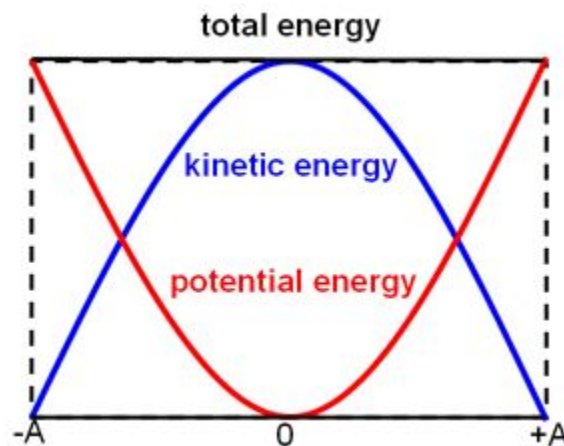
where m is the mass of the mass and k is the spring constant.

- Simple pendulum

- The period of the pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- Graph of energy against time



- **Dampening** - an oscillation is damped if dissipative forces remove energy to the system's surroundings, the types of damping are

- **Light damping** - The time period of each oscillation stays the same, the amplitude decreases gradually, reducing the same fraction each time
- **Critical Damping** - Is just enough to stop the system oscillating after being displaced. The system returns to the equilibrium in the shortest possible time without overshooting
- **Heavy Damping** - Occurs when the damping is so strong that the displaced object returns to the equilibrium much more slowly than it would normally

- **Forced Vibrations and Resonance**

- A periodic force is one that is applied at regular intervals
- Free vibrations are oscillations where the total energy stays the same over time
- Force vibrations occur when the object is forced to vibrate at a particular frequency by a periodic input of a force
- When an object is free to vibrate it will do so at its natural frequency

- When an object is forced to vibrate at its natural frequency it will have large amplitude vibrations - this is resonance
- The frequency with the largest amplitude is the resonant frequency
- The lighter the damping
 - The larger the maximum amplitude becomes at resonance
 - The closer the resonant frequency is to the natural frequency of the system
- As the frequency of the periodic force increases
 - The phase difference between the displacement and periodic force increase from 0 to $\frac{1}{2}\pi$
 - The phase difference is $\frac{1}{2}\pi$ at the maximum amplitude
 - The phase difference then increases from $\frac{1}{2}\pi$ to π as the frequency increases
- In an oscillating system with little damping at resonance
 - the applied frequency of the periodic force = the natural frequency of the system
- Stationary waves can be said to be in resonance

Gravitation

- Gravity is an attractive force that acts between all matter
- Newton's Law of Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

- A gravitational field is an area of 3D space where gravitational effects can be observed
- Field lines are used to represent the field, they are the path that a free 1kg mass would take
- **Gravitational field strength** is the gravitational force per unit mass
 - In a radial field it is

$$g = \frac{GM}{r^2}$$

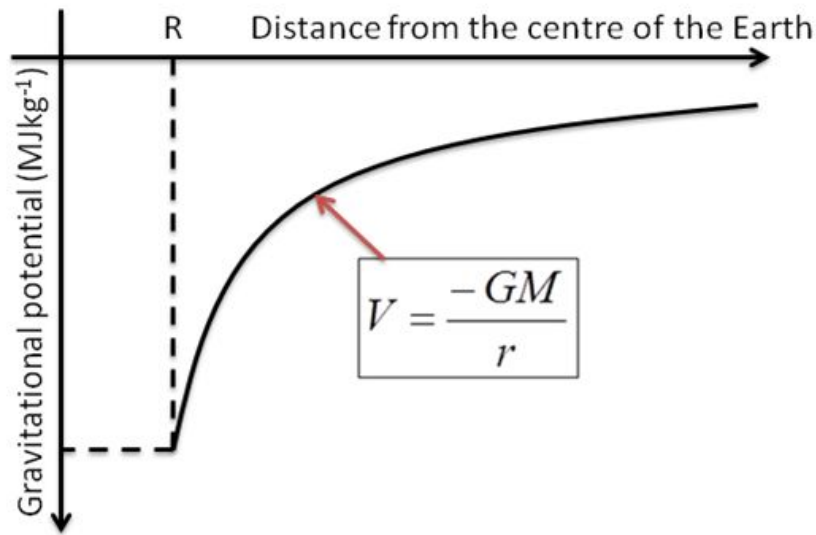
- **Gravitational Potential** is the work done to bring a 1kg mass from ∞ to a point in a gravitational field
 - The equation for this is

$$V = -\frac{GM}{R}$$

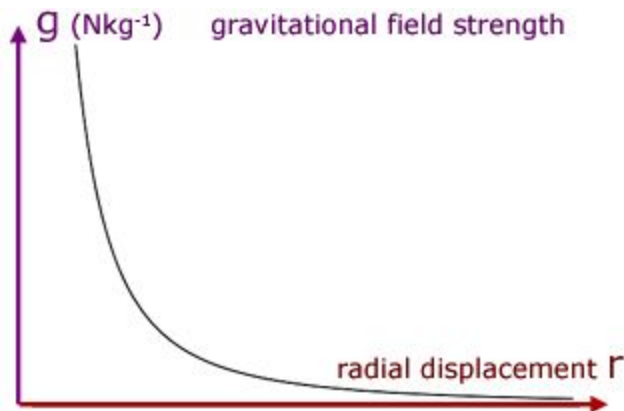
- **Gravitational Potential Energy** is the gravitational potential multiplied by the object's mass
- The work done in moving a mass is

$$\Delta W = m\Delta V$$

- Graph of Gravitational Potential



- Graph of Gravitation field strength



- Gravitational field strength is the negative of the differential of the graph of gravitational potential

$$g = -\frac{dV}{dr}$$

- The escape velocity is the speed required to completely escape the gravitational field of an object, given by

$$V_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$

- Kepler's Third Law

$$(\text{Period})^2 \propto (\text{Radius})^3$$

$$T^2 = \frac{4\pi^2}{GM} \cdot r^3$$

- Geostationary Orbits

- Orbit directly above the equator and have a time period of exactly 24 hours
- Remain in a fixed position above the equator
- Radius of a geostationary orbit = 4.2×10^7 m
- Are 3.6×10^7 m above the earth's surface

- Energy of an orbiting satellite
 - The total energy of a satellite is Potential Energy + Kinetic Energy
 - For a circular orbit

$$E = -\frac{GMm}{2r}$$

Electric Fields

- Coulomb's Law
 - The force between point charges in a vacuum

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{r^2}$$

- Electric Field Strength
 - is the force per unit charge
 - in a radial field is

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

- in a uniform field is

$$E = \frac{V}{d}$$

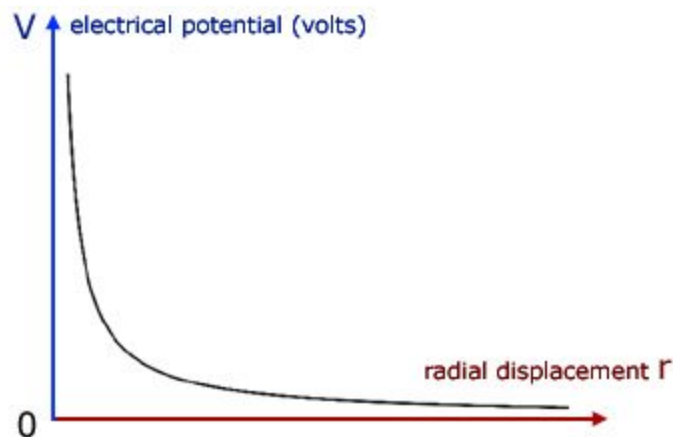
- Field lines are the path that a free +1C charge would take
- Electric Potential
 - is the work done to bring a +1C charge from infinity to a point in an electric field
 - in a radial field

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

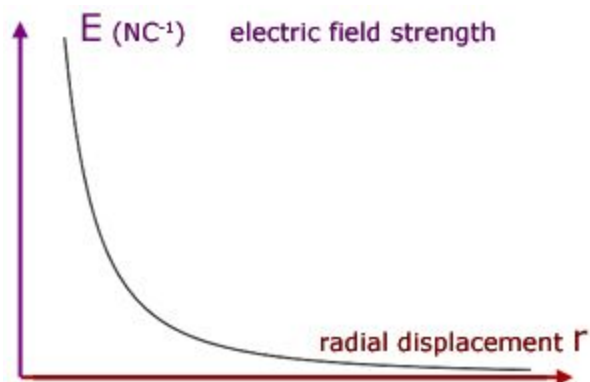
- The work done in moving a charge Q is given by

$$\Delta W = Q\Delta V$$

- Graph of electric potential



- Graph of electric field strength

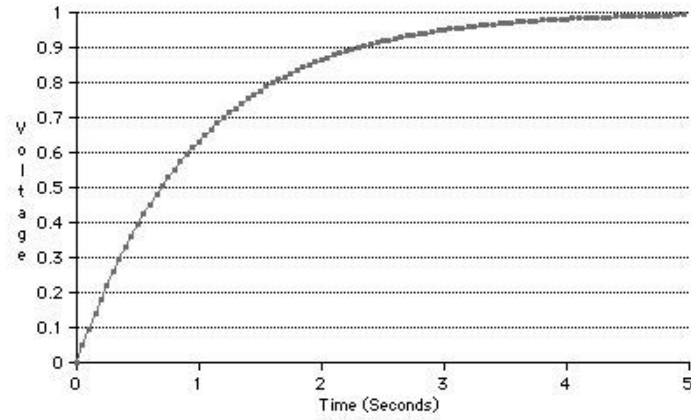


- Comparison of electric and gravitational fields
 - **Similarities**
 - Inverse square law of fields
 - Infinite range
 - **Differences**
 - Gravitational fields much weaker than electric fields
 - Gravity is only attractive while electric fields are also repulsive
 - Positive potential in electric fields while only negative in gravity

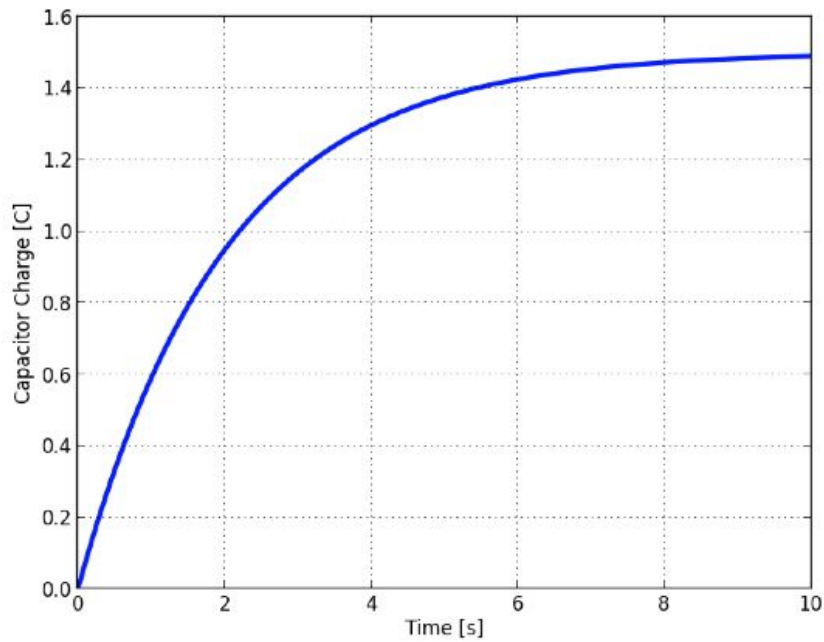
Capacitance

- Capacitance
 - The charge stored per unit of Pd of a capacitor
 - The unit of capacitance is the farad (F)
$$C = \frac{Q}{V}$$
- Energy stored in a capacitor
 - Area under a charge against pd graph is the energy stored
 - Because graph is triangular
$$E = \frac{1}{2}QV$$
 - This can be rewritten as
 - $E = \frac{1}{2}CV^2$
 - $E = \frac{Q^2}{2C}$

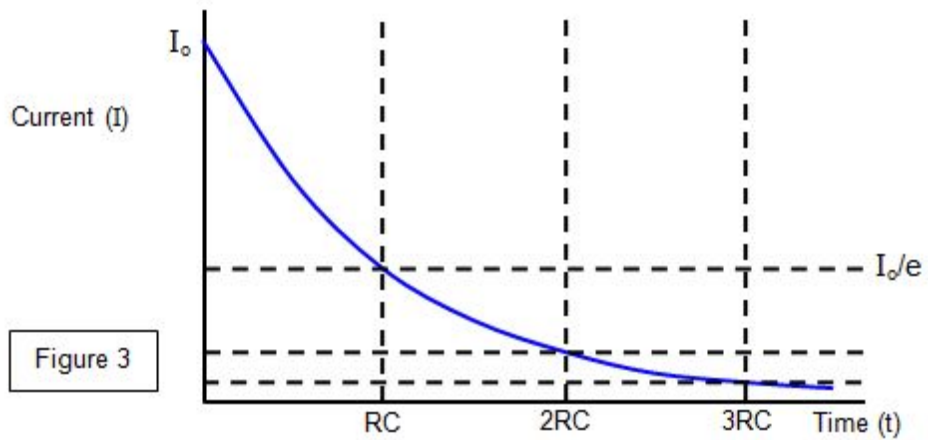
- Graph of capacitor charging
 - Voltage against time



- Charge against time



- Current against time



- Time Constant
 - This is the time taken for the charge to fall to 0.37 of its initial value when discharging
 - The time constant can be found using

$$t = RC$$

- The charge of a capacitor at any time during discharge is found using

$$Q = Q_0 e^{\frac{-t}{RC}}$$

- The voltage of a capacitor at any time during discharge is found using

$$V = V_0 e^{\frac{-t}{RC}}$$

Magnetic Fields

- Magnetic flux density
 - Is the amount of flux that passes through a metre squared of space at 90° to the area
 - The units of flux is the Weber (Wb)
 - Units are the Tesla (T) which is equal to Wb m^{-2}
 - Found using

$$B = \frac{\Phi}{A}$$

- When a current flows through a wire a magnetic field exists around it
- When a current moves through a wire in a magnetic field a force will act on the wire
 - The direction of the force is found using the left hand rule
 - The magnitude of the force is found using

$$F = BIl$$

$$F = BIl \sin \theta$$

- If a charged particle moves through a magnetic field it will experience a force
 - The direction of the force can be found using Fleming's left hand rule
 - The magnitude of the force is given by

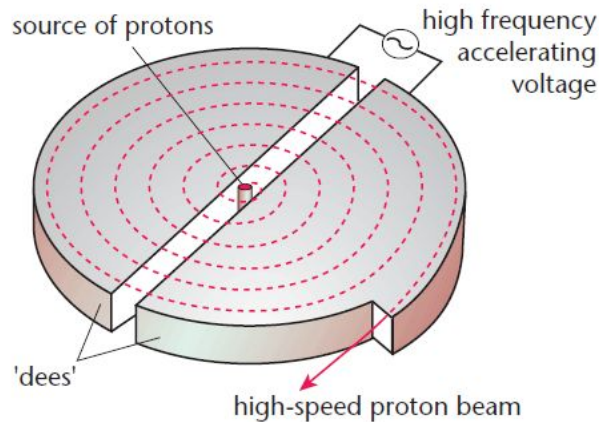
$$F = BQv$$

$$F = BQv \sin \theta$$

- Because the force is always at 90° to the path of the particle, it will point towards the centre of a circle. The path is **circular**
- Particles cannot gain kinetic energy
- The radius of the path is found by setting the force equation equal to that of centripetal force to give

$$r = \frac{mv}{BQ}$$

- Cyclotrons
 - Particle accelerators



- Function of the cyclotron
 - Charged particles are inserted into the centre of the cyclotron
 - The voltages accelerates the particles to one of the dees
 - The particles then move in a circle until it reaches the edge of the dee where the voltage accelerates the particle to the other dee
 - This continues until the particle is emitted
- The time of one circle is (a constant)

$$T = \frac{2\pi m}{BQ}$$

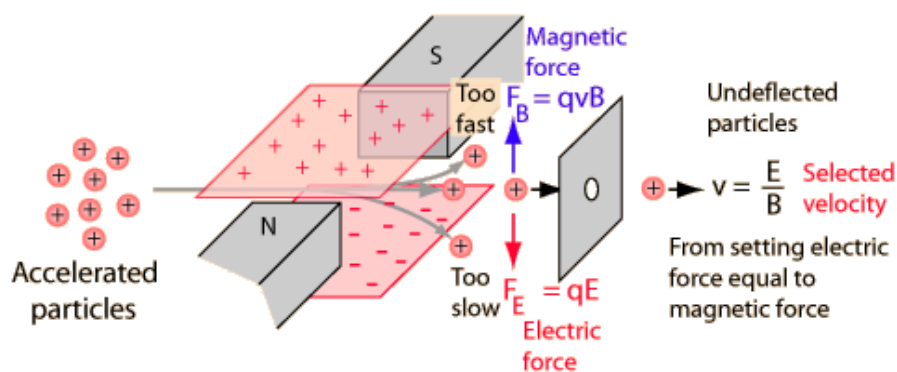
- The frequency of the current must be

$$f = \frac{BQ}{2\pi m}$$

- So that it accelerates the particle in the correct direction

- Mass Spectrometer

- The substance is ionised and accelerated using an electric field
- Only particles with the correct velocity pass through the velocity selector, which uses a magnetic and electric field at right angles to one another



- The electric force is not affected by velocity but the magnetic is so by changing the strength of the electric field velocities can be separated

- Different masses are deflected different amounts mass can be calculated as know velocity

- Flux Linkage

- is $N\Phi$, where N is the number of turns cutting the flux

- Can be written as

$$N\phi = BAN$$

- When you move a magnet into a coil a current is formed

- **Faraday's Law**

- The size of the induced current is directly proportional to the rate of change of the flux linkage
- This is written as

$$\xi = N \cdot \frac{d\phi}{dt}$$

- Increasing the velocity of the magnet increase the induced emf
- Increasing the number of coils increases the induced emf
- Increasing the strength of the magnetic field increases the induced emf

- **Lenz's Law**

- The direction of the induced emf is always in such a direction as to oppose the change which caused it

- The emf induced in a conductor as it moves through a magnetic field is

$$\xi = Blv$$

- For a rectangular coil moving into a uniform magnetic field the emf generated is

$$\xi = BNlv$$

where l is the length of the side parallel to the edge of the field

- **AC Generators**

- The flux linkage at an angle θ between the normal of the coil's plane and the magnetic field is

$$N\phi = BAN\cos\theta$$

- The induced emf at time t is

$$\xi = BAN\omega\sin\omega t$$

where ω is the angular velocity

- **Transformers**

- Operation

- AC in the primary coil creates alternating flux in the primary
- Common iron core transmits the flux to the secondary
- Changing flux in the secondary coil generates an AC

- The transformer equation

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

- If 100% efficient

$$V_p I_p = V_s I_s$$

- Causes of inefficiency

- Resistive heating in the wires
- Loss of magnetic flux
 - Core must have high permeance
 - Make core thick and short to increase permeance
- Eddy currents cause core heating
 - `lar currents form to counter flow of flux, Lenz's Law

- Large eddies form if the core is large, lamina used to reduce the size and therefore heating effect of eddies